14 Schlag den Staab

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Challenge

Mathilda is thrilled: her favorite show, "Schlag den Staab", is returning to television.
In the show, a contestant competes against the TV star Evan Staab in 100 games.[1] Each game is won either by the contestant or Evan Staab; a tie is not possible. The winner of the first game earns one point, the second game two points, and so on, so the winner of the final game earns 100 points. These points are added together, and whoever has the most points at the end wins the show. If the contestant and Evan Staab have the same total points at the end, a tiebreaker game is held.
Mathilda loves it when a tiebreaker happens, as it makes the show particularly exciting. After every game, the current standings are displayed, and Mathilda wonders whether it is still mathematically possible for the final score to result in a tie. Sometimes, a tiebreaker remains possible until the very last game, but more often it becomes clear much earlier that a tiebreaker is no longer possible.
Let k be the smallest positive integer such that, after the k-th game, it may no longer be possible for the final score to result in a tie. What is the last digit of k in the decimal system?

Hint:
The Gaussian summation formula is

$$1 + 2 + 3 + \ldots + (n-1) + n = \frac{n(n+1)}{2}.$$

(*Possible answers on next page*)

[1] In the German version, there are only 15 games, but since a polar night at the North Pole lasts six months, late-night shows are correspondingly longer...

Possible Answers:

1. 1
2. 2
3. 3
4. 4
5. 5
6. 6
7. 7
8. 8
9. 9
10. 0