



8 Wild Reindeer

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Project: EF5-2

Challenge

Santa Claus has acquired new wild reindeer that he would like to use. However, he lacks the time to train them and has only limited knowledge about their movement. He knows that the position x_t of the reindeer at time step t and the velocity v_t at time step t follow some rules. To describe these rules properly, he introduces a coordinate system on the ground (plane). Then he can write down iterative equations for the velocity v_t and position x_t :

$$v_{t+1} = -0.5v_t - ax_t$$

$$x_{t+1} = x_t + v_{t+1},$$

where the so called fear value a is a positive real number that depends on the reindeer. The fear value describes whether a reindeer is tame and will eventually stop running away from Santa. If a reindeer's fear value a is favorable, the reindeer will become arbitrarily slow and will remain slow over time. Santa can then catch it and bring it home. However, there are some reindeer that cannot overcome their fear. These reindeer keep running faster and faster and cannot be caught.

We are looking for all fear values a that a reindeer can have so that Santa can catch it. It is assumed that the reindeer start with an initial velocity $v_0 = (0, 0)$ at the position $x_0 = (1, 1)$.

Hints and remarks:

Note, that the multiplication of a number λ and a point (x_1, x_2) is defined by $\lambda(x_1, x_2) = (\lambda x_1, \lambda x_2)$. The addition or subtraction is defined coordinate wise: $(x_1, x_2) \pm (y_1, y_2) = (x_1 \pm y_1, x_2 \pm y_2)$.

To solve this challenge it can be helpful to make use of the absolute value. The absolute value represents the distance of a number to 0 and is defined by

$$|x| = \begin{cases} x & , \text{ if } x \geq 0 \\ -x & , \text{ if } x < 0 \end{cases}.$$

For the absolute value and two real numbers a, b , the following rules hold:

$$|a + b| \leq |a| + |b|$$

$$|a - b| \geq ||a| - |b||$$

$$|ab| \leq |a||b|$$

The term “become and remain arbitrarily slow” in this challenge is meant as follows: For any positive number $\epsilon > 0$ there is a time t_0 such that if $t \geq t_0$, then each component v of the velocity v_t fulfills $|v| < \epsilon$.

Possible Answers

1. $0 < a < 0.1$
2. $0 < a < 0.2$
3. $0 < a < 0.3$
4. $0 < a < 0.4$
5. $0 < a < 0.5$
6. $0 < a < 0.6$
7. $0 < a < 0.7$
8. $0 < a < 0.8$
9. $0 < a < 0.9$
10. $0 < a < 1$

Project Reference

In the EF5-2 project “Data-driven Modeling of the Romanization Process of Northern Africa,” historical data is used to infer networks that represent the spread of culture. Due to the sparsity of the data, many networks provide similar explanations for the spreading process. Therefore, it is crucial to use algorithms that can explore a wide range of possible solutions. One such algorithm is Particle Swarm Optimization (PSO), which simulates the movement of a bird swarm, with the positions and velocities of the particles governed by a difference equation, like the one shown here. Different parameters can lead to different dynamic behaviors of the algorithm, and these variations can be beneficial depending on the specific problem.