The Olympic Housing Dilemma

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Challenge

This year, the Olympic Arctic Games are held at the North Pole, attracting 1,024 athletes from across the Frozen North to compete in various sports. Otto, the event's accommodation organizer, is tasked with housing the athletes. Therefore, he needs to follow one strict rule of the Arctic Olympic Committee: If two (distinct) athletes do not share a common language, they cannot be accommodated together in the same house. To comply with this rule, Otto asks each athlete in advance which languages they could speak.

Otto makes some interesting observations:

- There are exactly 10 languages (L_1 to L_{10}) that are spoken.
- Every possible combination of these 10 languages is spoken by exactly one athlete.
  This means, for example, that there is exactly one athlete who speaks only the language L_1, and similarly, there is exactly one athlete who speaks only the languages L_2, L_3, and L_8, and so on.
- Thus, there is also exactly one athlete who speaks all 10 languages, and exactly one athlete who speaks no language at all.

Of course, Otto must also work as efficiently as possible. He wants to build as few houses as possible while still adhering to the committee's rules. How many houses does Otto need to build at a minimum to accommodate all 1,024 athletes? Let k represent this minimum number of houses. What is the last digit of k in the decimal system?

(*Possible answers on next page*)

Possible Answers:

1. 1
2. 2
3. 3
4. 4
5. 5
6. 6
7. 7
8. 8
9. 9
10. 0

Project Reference:

This puzzle is an example of a coloring problem and while these are particularly fun, they are usually difficult. Usually we have a certain set V whose elements we would like to color; however, the devil is allowed to give us an arbitrary list of conditions E detailing pairs that are not allowed to be colored the same. We are usually interested in determining the least possible number of colors sufficient to paint the elements of our sets without violating any of the devil's conditions. This pursuit is proved to be notoriously challenging; in fact, there is no known fast way to determine whether an arbitrary set V with a corresponding list E of conditions can be properly colored with only 3 colors. If you find a fast algorithm for that purpose, you should hire a secretary as many people would love to talk with you.

This particular problem has a pretty famous cousin: Suppose there are n languages at the north pole where every athlete speaks exactly k languages and every subset of k languages is spoken by exactly one athlete. One can now ask again for the smallest number of houses needed to accommodate the athletes so that no two with no language in common share the same house. This problem was open for 22 years before it was solved using seemingly unrelated theorems from topology!