



## 7 Santa Cargo

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### Challenge

The company SANTA CARGO is responsible for shipping all the presents. Of course, SANTA CARGO wants to minimize the effort in order to deliver all gifts on time and in the optimal way. Can you help the company?

For the next shipment SANTA CARGO has to send  $N = 10$  gifts. Traditionally, they use sleds to transport the cargo. A sled always comes with an initial effort of one elf-watt (the standard unit of measuring effort at the North Pole). Moreover, there is an additional effort depending on the number of gifts.

A sled can carry up to 10 gifts, but it is best balanced to carry three gifts. Depending on the number of gifts  $n$ , the effort is given by  $(n - 3)^2$  elf-watt. This means that a sled carrying only one gift takes more effort than a sled loaded with three gifts. That is because SANTA CARGO wants to prevent underloaded or even empty sleds. So the effort by a single sled carrying  $n$  gifts can be calculated by:

$$e(n) := 1 + (n - 3)^2.$$

How many sleds should SANTA CARGO load and how should the presents be divided up in order to ship the ten gifts with minimum effort?

*(Possible answers on the next page)*

A possible answer should have the form  $(n_1, \dots, n_k)$  where  $k$  is the number of sleds and  $n_i$  is the number of gifts on sled  $i$ . So, for example, the answer (10) has one sled carrying all the gifts, but  $(2, 5, 3)$  has three sleds, the first one loaded with two gifts, the second has five gifts and the third carries three gifts. Also, note that a gift cannot be cut into several pieces.

**Possible answers:**

1. (10)
2. (10, 0)
3. (5, 4, 3)
4. (2, 3, 2, 3)
5. (2, 2, 2, 2, 2)
6. (7, 3)
7. (4, 4, 2)
8. (1, 2, 3, 4)
9. (5, 5)
10. None of the above. The right answer is ...