



6 Workbench Hubbub

Author Tobias Paul (HU Berlin)

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Challenge

In order to wrap the Christmas presents appropriately, Father Christmas has provided wrapping paper in the colours red and blue this year.

A total of 500 Christmas elves sit at a workbench with their presents to be wrapped in pairs facing each other (i.e. 250 on each side). The elves proceed as follows:

- Each elf wraps their present at the same time as the elf sitting opposite.
- The pair of elves at the start of the workbench wrap their presents first. Each elf decides for themselves whether to use red or blue wrapping paper.
- Then the next pair of elves sitting opposite each other wraps their presents, followed by the third pair and so on. Wrapping in pairs therefore starts at one end of the workbench and gradually continues to the other end.

On day 1, each elf sees the packaging of the previous elf on the opposite side out of the corner of their eye and decides to use exactly the same colour for the wrapping paper.

The first pair decides on one of the two colours with a probability of $1/2$, completely independently of each other.

On day 2, each elf sees the packaging of the previous 2 elves on the opposite side out of the corner of their eye. If the two packages are the same colour, they will choose the same colour. However, if he sees different colours, he

will choose one of the two colours completely independently with a probability of $1/2$. The first 2 pairs at the beginning of the table decide completely independently of each other with a probability of $1/2$ for the colour of their wrapping paper.

Shortly before Christmas, Santa is now faced with the following questions: What is the probability that both elves of the last pair of elves (elves number 250) at the table will use the same colour for their wrapping paper (a) on day 1 and (b) on day 2?

Watch out, Father Christmas has rounded the possible answers to the 2nd decimal place.

Note: For day 2, for example, it could be helpful to think about the possibilities for the first five pairs.

Possible Answers:

1. (a): 1 (b): 1
2. (a): 1 (b): 0.5
3. (a): 1 (b): 0
4. (a): 0.5 (b): 1
5. (a): 0.5 (b): 0.5
6. (a): 0.5 (b): 0.33
7. (a): 0.5 (b): 0
8. (a): 0 (b): 1
9. (a): 0 (b): 0.5
10. (a): 0 (b): 0.33

Project Reference:

In game theory, there is a so-called “adaptive play” process. In the specific case of the coordination game with two players and two options (in which both players want to agree, i.e. choose the same option), this process does

the following: To make the decision for the next game round, each player is allowed to look at a number $s \geq 1$ of the opponent's moves from the last h game rounds. In the task, this corresponds to $s = 1$ and $h \in \{1, 2\}$. The best response to what is seen in this game is then to copy the move (because the players want to match).

In the project, we want to investigate variants of this process using methods from population genetics and thus make possible statements about e.g. frequencies of moves over time.