5 Coating cookies with glaze

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Challenge

The elves Bob and Sieglinde were sitting happily playing chess on Advent Day. Suddenly the clock struck, heralding their shift at the Christmas factory. Their task in the Christmas factory is to coat cookies, consisting of 10 square pieces, of the rectangular shapes 1×10 and 2×5 with glaze (see figure 1).

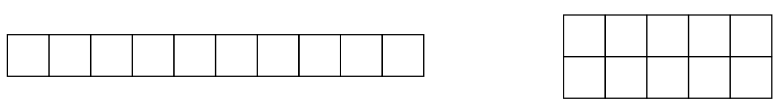

Figure 1: The cookies of the shapes 1×10 (left) and 2×5 (right).

To ensure that the cookies remain unique in their decoration, Bob and Sieglinde coat pieces of the cookies with glaze according to the “Christmas chess principle”. Like chess kings hitting the up to eight squares around them on a chess board, they only apply the glaze to the individual squares in such a way that no two squares covered with glaze touch each other on an edge or corner. At the same time, however, there must be no space left for another glazed square on the entire cookie. Optimization is not an issue here. It is therefore quite possible that in one arrangement more and in another fewer glaze squares would have fitted on the entire rectangular cookie.

How many different ways are there to glaze the cookies in the shapes 1×10 and 2×5? The answer must be given in pairs of numbers (M, N), where M

stands for the shape 1×10 and N for the shape 2×5.

Note: Consider only the final configurations as different. Moreover, configurations that can be transformed into each other by rotating the cookie are also considered as different.

Possible answers:

1. $(9, 12)$
2. $(12, 12)$
3. $(12, 16)$
4. $(12, 20)$
5. $(12, 24)$
6. $(12, 36)$
7. $(16, 16)$
8. $(16, 20)$
9. $(16, 24)$
10. $(16, 36)$

Project reference:

This MATH$^+$ project focuses on biochemical oscillators. In particular, the characteristic intracellular conditions imply that chemical reactions take place in very small volumes. At these microscopic scales, the discrete nature of the reacting components becomes crucial. This means that the biochemical processes taking place are inherently stochastic and classical macroscopic models of reaction kinetics are often not applicable. The proposed challenge of counting configuration possibilities is a simple example that borders directly on the field of stochastics and probability theory. The exercise shows that even in relatively simple situations, direct counting of possibilities can become difficult. However, by skillfully using mathematical methods, the effort can be considerably reduced and more general solutions can be obtained.