11 Go for Christmas!

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Challenge

In order to pass the time on a dark rainy day before Christmas, the three dwarfs Godric, Goliath and Gowan take up the challenge to fill up the 18×18 squares of a traditional Go board with white and black stones in an artistic and visually attractive way. To reach their goal they agree to use the following basic blocks for their designs:

1. Bricks consist of a single-colored strip exactly one row wide, surrounded by a frame of the other color. The notation k-brick for $k \geq 1$ describes a brick whose single-colored strip has a length of k.

2. Tiles consist of a single-colored square in the center, surrounded by a frame of the other color. The notation k-tile for $k \geq 1$ describes a tile whose central single-colored square has a size of $k \times k$.

For example, a white 1-brick, a white 2-brick, and a black 4-brick are shown on the left in Figure 1. Similarly, a black 1-tile, a black 2-tile, and a white 4-tile are shown on the right in Figure 1.
A 1-brick and a 1-tile of the same color are therefore identical.

The challenge of the three dwarfs is to use the basic blocks to fill up the 18×18 squares of a traditional Go board with white and black stones in such a way that:

- all the squares are covered;

- the total number of used white stones is equal to the total number of used black stones;

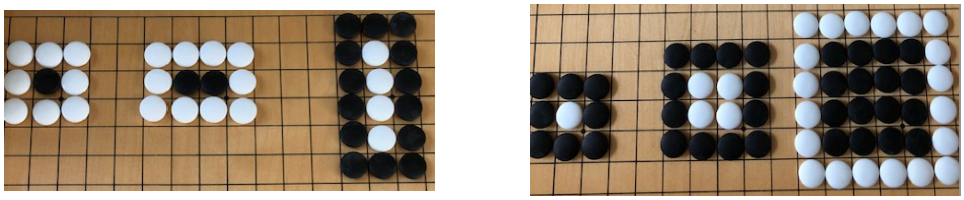

Figure 1: Representation of bricks and tiles. Left: A white 1-brick, a white 2-brick, and a black 4-brick. Right: A black 1-tile, a black 2-tile, and a white 4-tile.

- Two different basic blocks with the same frame color do not touch, but maybe adjacent at the corners.

Figure 2 shows a partial cover that is allowed (left upper corner) and a partial cover that is not allowed (right lower corner; the red dots indicate a conflict).

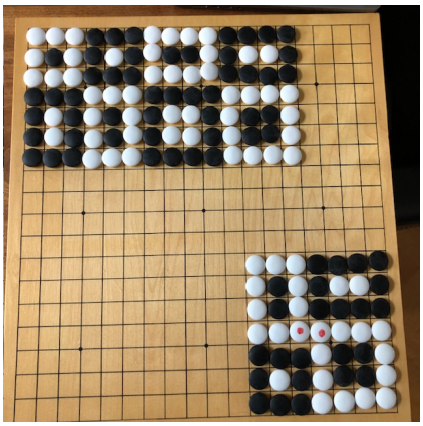

Figure 2: An allowed (upper left) and a disallowed (lower right) partial cover

The dwarves now aim to use as many basic blocks of different sizes as possible. They refer to the number of basic components of varying sizes as the aesthetic quality. Colors are ignored for this count. As an example: The partial covering in the upper part of Figure 2 uses 1-bricks (1-tiles), 2-bricks, 2-tiles, and no other basic components. Therefore, this partial covering of a 7×14 board would receive a score of 3 for aesthetic quality.

The three dwarfs examine different strategies for designing proper covers. Hereby,

- Godric is only allowed to use 1-bricks, 2-bricks, 3-bricks etc., but no k-tiles with $k \geq 2$
- Goliath is only allowed to use 1-tiles, 2-tiles,3-tiles etc., but no k-bricks with $k \geq 2$
- Gowan is free to use a mixture of any of the bricks and tiles.

Each of them tries to obtain the highest possible score, respecting all the restrictions.
Question: What are the highest possible scores of Godric, Goliath and Gowan?

Possible answers:

1. The highest scores of Godric, Goliath and Gowan are 3, 1 and 3, respectively.
2. The highest scores of Godric, Goliath and Gowan are 3, 1 and 4, respectively.
3. The highest scores of Godric, Goliath and Gowan are 3, 2 and 5, respectively.
4. The highest scores of Godric, Goliath and Gowan are 4, 1 and 4, respectively.
5. The highest scores of Godric, Goliath and Gowan are 4, 1 and 5, respectively.
6. The highest scores of Godric, Goliath and Gowan are 4, 2 and 6, respectively.
7. The highest scores of Godric, Goliath and Gowan are 5, 1 and 5, respectively.
8. The highest scores of Godric, Goliath and Gowan are 5, 2 and 5, respectively.
9. The highest scores of Godric, Goliath and Gowan are 5, 2 and 6, respectively.
10. None of the other possible answers is correct.