9 Christmas Logistics

Authors: Marieke Heidema & Amir Shakouri (University of Groningen)

Challenge

Every year, Santa's elves keep track of the number of people that live in a place. That way, they know exactly how many presents to make for each town and city. They write down this number in a book. This book also contains other information about the place, like the location and the weather forecast for that place.

When they check the book shortly before Christmas, they cannot find the population of a small island in the Atlantic. The elves start to panic. "What do we do now," they ask each other. A hasty discussion takes place. "Can we not just quickly go to the island and count the number of inhabitants again?" asks young Beatrice. "No time, no time!" answers Theo. Concerned, elf Leonard asks "So then we have to guess how many presents we need, but what if we don't bring enough presents?" Matthew, one of the senior elves, adds to this "Well, we can bring some extra presents, but we can surely not bring too many presents. That will make Santa's sled too heavy to pull for the reindeers. It will slow the sled down and the presents may not be delivered on time!" "So what can we do then?" Beatrice asks, "We cannot bring too few, nor way too many. So how many do we bring?"

The elves fall silent, they think and look around at each other, hopeful. But no one seems to know what to do. Then, a soft cough comes from the doorstep, it's Lennard, a young elf. It's his first year helping with the Christmas preparations and he stayed quiet until now. When he coughs, the other elves look around at him. "Is there anything you want to say, Lennard? Do you have an idea?" Theo asks. In a soft but confident voice,

Lennard answers "Yes, I think I know what to do. Let me explain..."

Lennard points to the page in the book with the island's data. There is an old note about the island there, which reads as follows:

Christmas 2022 : ***Attention, new island, don't forget to count the inhabitants!***
Note, there is an ongoing project aiming to create new spaces for people to live. Islands that were not inhabited before are now being inhabited. This particular island has space for 200 people but is initially only inhabited by 25 people. The increase rate with which the population will grow is thought to be 2.

Lennard pushes his round glasses further on his nose and starts to explain: "Look, we can use mathematics to estimate how many people live on the island now." He writes the following on a blackboard:

$$P(t+1) - P(t) = \left(r - \frac{r}{C}P(t)\right) P(t).$$

"Here, $P(t)$ is the size of the population in year t. The note tells us that the initial population size in 2022 was 25, meaning that $P(2022) = 25$. Furthermore, r stands for the approximate rate of increase of the population. By the note we know that $r = 2$. Lastly, C is the capacity of the island, which we know by the note is $C = 200$."

Based on this, what will be the estimated number of inhabitants in Christmas 2024?

Possible answers:

1. 0 inhabitants.
2. 1 to 22 inhabitants.
3. 23 to 55 inhabitants.
4. 56 to 71 inhabitants.
5. 72 to 98 inhabitants.
6. 99 to 123 inhabitants.
7. 124 to 150 inhabitants.
8. 151 to 183 inhabitants.
9. 184 to 199 inhabitants.
10. 200 inhabitants.